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Vacuum states in 2D tachyon effective action

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Abstract: In this paper we will study ground states of the toy model of 2D closed string tachyon effective action. We will firstly construct the classical solutions of the tachyon effective action that do not induce backreaction on metric and dilaton. Then we will study the quantum mechanics of the zero mode of the tachyon field. We will find family of vacuum states that are labeled with single parameter. We will also perform the quantum mechanical analysis of the tachyon effective action when we take into account dynamics of nonzero modes. We will calculate the vacuum expectation values of components of the stress energy tensor and dilaton source and we will argue that there is not any backreaction on metric and dilaton.

Keywords: Bosonic Strings, Tachyon Condensation.

Contents

1. Introduction

To find off-shell description of closed string is very difficult task. The most interesting exception is noncritical string theory in two dimensions.[1] Here, there is a complete non-perturbative description in terms of the double scaling limit of matrix quantum mechanics. For matrix quantum mechanics the space of eigenvalues of the matrix provides the space dimensions on which free string moves. In this case, however, the string theory is rather trivial: for a bosonic string the only dynamical degree of freedom is a single massless scalar which is related to the collective field-the density of eigenvalues. The eigenvalues of this theory behave as fermions and the collective field may be regarded as a bosonisation of these fermions. In summary, the discovery of the double scaling limit of $c < 1$ matrix models provided beautiful and exact solution to the bosonic string theory to $D \leq 2$ spacetime dimensions. However one disappointing property of these models was the fact that bosonic string theory did not appear to be well defined non-perturbativelly. Recently this problem appears to have been removed by reinterpretation of these models as nonperturbatively well defined type 0 strings[2] [9, 10].

The two dimensional (2D) bosonic theory is defined in the linear dilaton background (In units, where $\alpha' = 1$)[3]

$$g_{\mu\nu} = \eta_{\mu\nu} , \qquad \Phi = 2x^1 . \tag{1.1}$$

It is well known that dilaton is generator of the string coupling in the sense $g_s \sim e^{\Phi}$ so that for the solution (1.1) there is a region where the coupling constant diverges and string perturbative theory fails. This means that such a background does not define satisfactory

[1] For reviews, see [1]–[8].

[2] For extensive list of references, see excellent recent review [8].

[3] For review, see [11].

string theory. It turns out however that this can be cured by inclusion of the tachyon condensate in the form

$$T = \mu e^{2x^1} , \tag{1.2}$$

where μ is "cosmological constant". The claim is that (1.1) and (1.2) defines exact string theory background. Indeed, in this background the string sigma model takes the form

$$S = \frac{1}{4\pi} \int d^2\sigma \sqrt{h} \left[h^{ab} \partial_a X^\mu \partial_b X_\mu + 2\mathcal{R} X^1 + \mu e^{2X^1} \right] . \tag{1.3}$$

It can be checked that this action represents exact conformal field theory and hence defines consistent string theory.

It is certainly important to find an action that has background fields (1.1), (1.2) as its exact solution. This intuition is based on the fact that (1.3) defines exact CFT and hence β functions that we associate with each field in the worldsheet theory vanish. Then it is believed that these equations arise from variation of some spacetime effective action. However to find such effective actions from the first principles of string theory is very difficult task. Another possibility is to try to guess their form from the requirement that the background that defines exact conformal field theory is solution of the equation of motions that arises from it. Such approach was very useful for the searching of the tachyon effective actions in case of unstable D-branes in string theories [12]–[15] that has marginal tachyon profile as its exact solution.[4] One can then ask the question whether similar approach could be applied in the case of closed string tachyon effective action. In fact, preliminary steps in this direction were given recently in [18]. The basic property of the proposed closed string tachyon effective action given there is that it has the marginal tachyon profile $T = \mu e^{\beta_\mu x^\mu}$ in the linear dilaton background as its exact solution. We have also shown that at two dimensions the situation simplifies considerably and we were able to find field theory effective action for tachyon, dilaton and metric that has the linear dilaton background (1.1), (1.2) as its exact solution even if we take into account the backreaction of the tachyon on metric and dilaton.

In this paper we will continue the study of this model of 2D tachyon effective action. Namely, we focus on the ground states of this action, either from classical or quantum point of view. As we will demonstrate in the next section the tachyon effective action has remarkable property that if we begin with the tachyon background with some cosmological constant μ_1 in (1.2) then we can easily reach the other tachyon background (1.2) characterised with the second cosmological constant μ_2. We hope that this is sign of the background independence of the tachyon effective action. At first sight it seems that this is rather trivial fact from the point of view of the tachyon effective field theory. On the other hand we mean that such a form of background independence is not directly visible in the fundamental formulation of two dimensional string theory, either in free fermion description or in the collective field theory description. For that reason we believe that

[4]We must also stress that it is not currently clear how such tachyon effective actions are related to these that were calculated from the first principles of the string theory [16]. For more detailed discussion of this issue, see [17].

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the study of the background independence in the tachyon effective action can be useful for understanding of the fundamental properties of the microscopic formulation of 2D string theory.

Since generally the free fermion formulation of the two dimensional string theory is quantum theory and the parameter μ appears in the definition of the Fermi sea it seems to be natural to study the ground states in the quantum mechanical formulation of the tachyon effective action. We will see that there exists family of vacuum states labelled by single parameter that is directly related to μ in (1.2). We will also calculate the expectation values of the stress energy tensor and dilaton source in these states and confirm that there is not any backreaction on metric and dilaton in agreement with classical analysis. We will also construct an operator that maps one vacuum state to another one. We think that the existence of this operator confirms the presumption that all ground states with different values of μ are equivalent.

We must also stress one important limitation of our analysis. As is well known from the effective field theory description of the unstable D-branes the validity of the tachyon effective actions strongly depends on the region in the field theory space where they are defined. Then it is clear that the model of the tachyon effective action that was proposed in [18] cannot describe the full dynamics of the closed string tachyon. Rather we should restrict to the modes that propagate very close to the classical solution (1.2). However we still believe that conclusions considering the ground states of the tachyon effective action are more general and could be helpful in the study of general properties of the tachyon dynamics.

The structure of this paper is as follows. In the next section (2) we give the brief review of the tachyon effective action that was suggested in [18]. In section (3) we study the quantum mechanics of the zero mode of the tachyon field. In section (4) we extend this analysis to the full quantum field theory of the tachyon effective action and we calculate the vacuum expectation values of the stress energy tensor and dilaton source. In conclusion (5) we outline our results and suggest further extension of this work.

2. Toy model of 2D tachyon effective action

In this section we review basic facts about the toy model of 2D closed string tachyon effective action suggested in [18].

As it is well known spacetime fields of two dimensional bosonic string theory consist dilaton, graviton and tachyon. The dilaton and graviton are described with the action[5]

$$S_{g,\Phi} = -\int d^2x\sqrt{-g}e^{-2\Phi}\left(16 + R + 4g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right). \tag{2.1}$$

In [18] we have proposed the tachyon effective action that has the marginal tachyon profile $T = \mu e^{2x^1}$ in the linear dilaton background $\Phi = 2x^1$ as its exact solution even if we take

[5]Our convention is as follows. We work with two dimensional metric with signature $(-,+)$. The spatial coordinate is labelled with $x^1 \equiv \phi$ and time coordinate is $x^0 \equiv t$.

into account the backreaction of the tachyon on the dilaton and the graviton. This action has the following form

$$\begin{aligned} S_T = & -\int d^2x \frac{\sqrt{-g}}{(1+e^{-2\Phi}T^2(4-g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi))} \times \\ & \times\sqrt{1+e^{-2\Phi}\left(4T^2+g^{\mu\nu}\partial_\mu T\partial_\nu T-2Tg^{\mu\nu}\partial_\mu T\partial_\nu\Phi\right)}+ \\ & +\int d^2x\sqrt{-g}\frac{1}{1+\frac{1}{2}T^2e^{-2\Phi}(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi)}\,. \end{aligned} \tag{2.2}$$

The variation of the action $S = S_{g,\Phi} + S_T$ with respect to $g^{\mu\nu}$ gives the equation of motion for $g_{\mu\nu}$

$$e^{-2\Phi}\left(G_{\mu\nu}-2g_{\mu\nu}\nabla^2\Phi+2\nabla_\mu\nabla_\nu\Phi+2g_{\mu\nu}\left(\nabla\Phi\right)^2-8g_{\mu\nu}\right)=T^T_{\mu\nu} \tag{2.3}$$

and the variation with respect to Φ gives

$$\sqrt{-g}e^{-2\Phi}\left[32+2R-8g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right]+16e^{-2\Phi}\partial_\mu\left[\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right]=J_\Phi\,, \tag{2.4}$$

where

$$T^T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta S_T}{\delta g^{\mu\nu}}\,,\qquad J_\Phi=\frac{\delta S_T}{\delta\Phi}\,. \tag{2.5}$$

The explicit forms of the stress energy tensor and dilaton source are

$$\begin{aligned} T_{\mu\nu} = & -g_{\mu\nu}\frac{1}{(1+e^{-2\Phi}T^2(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi))}\sqrt{\mathbf{B}}+ \\ & +\frac{2e^{-2\Phi}T^2\partial_\mu\Phi\partial_\nu\Phi}{(1+e^{-2\Phi}T^2(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi))^2}\sqrt{\mathbf{B}}+ \\ & +\frac{e^{-2\Phi}(\partial_\mu T\partial_\nu T-T\left(\partial_\mu T\partial_\nu\Phi+\partial_\mu\Phi\partial_\nu T\right))}{(1+e^{-2\Phi}T^2(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi))}\sqrt{\mathbf{B}}- \\ & -g_{\mu\nu}\frac{1}{1+\frac{1}{2}T^2e^{-2\Phi}(4-g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi)}-\frac{e^{-2\Phi}T^2\partial_\mu\Phi\partial_\nu\Phi}{(1+T^2e^{-2\Phi}\frac{1}{2}(4-g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi))^2}\,, \end{aligned} \tag{2.6}$$

$$\begin{aligned} J_\Phi = & -\frac{2\sqrt{-g}e^{-2\Phi}T^2(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi)\sqrt{\mathbf{B}}}{(1+e^{-2\Phi}T^2(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi))^2}+ \\ & +\partial_\mu\left[\frac{2\sqrt{-g}e^{-2\Phi}T^2g^{\mu\nu}\partial_\nu\Phi\sqrt{\mathbf{B}}}{(1+e^{-2\Phi}T^2(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi))^2}\right]+ \\ & +\frac{\sqrt{-g}e^{-2\Phi}(4T^2+g^{\mu\nu}\partial_\mu T\partial_\nu T-2Tg^{\mu\nu}\partial_\mu T\partial_\nu\Phi)}{(1+e^{-2\Phi}T^2(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi))\sqrt{\mathbf{B}}}- \\ & -\partial_\mu\left[\frac{\sqrt{-g}e^{-2\Phi}Tg^{\mu\nu}\partial_\nu T}{(1+e^{-2\Phi}T^2(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi))\sqrt{\mathbf{B}}}\right]+ \\ & +\sqrt{-g}\frac{e^{-2\Phi}(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi)}{(1+\frac{1}{2}T^2(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi))^2}-\partial_\mu\left[\frac{\sqrt{-g}T^2e^{-2\Phi}g^{\mu\nu}\partial_\nu\Phi}{(1+\frac{1}{2}T^2(4-\partial_\mu\Phi g^{\mu\nu}\partial_\nu\Phi))^2}\right]. \end{aligned} \tag{2.7}$$

As we have shown in [18] the equations of motions (2.3) and (2.4) have following exact solution corresponding to the linear dilaton background

$$\Phi=V_1x^1\,,\qquad V_1=2\,,\qquad g^{\mu\nu}=\eta^{\mu\nu} \tag{2.8}$$

together with the nonzero tachyon profile

$$T = \mu e^{2x^1} \tag{2.9}$$

that is also solution of the equation of motion that arises from the action (2.2) if we take into account the ansatz (2.8) [18]. According to the standard dictionary we can interpret (2.9) as the vacuum expectation value of the tachyon field T

$$\left\langle T(x^1, t) \right\rangle = \mu e^{2x^1} . \tag{2.10}$$

At this place we must stress one important point considering the vacuum expectation value of the tachyon field T [4]. As was argued for example in [19, 20] the correct vacuum expectation value of T, rather than given in (2.10), should be equal to[6]

$$T(x^1) = 4g_s^{-1}(2x^1 + c)e^{2x^1} , \qquad c = 1 + \Gamma'(1) + \ln g_s , \qquad g_s^{-1} = \frac{|\mu|}{\sqrt{2\pi}} , \tag{2.11}$$

where g_s is string coupling constant and μ is level of Fermi sea. From the point of view of the tachyon effective action (2.2) one can understand the discrepancy between (2.10) and (2.11) as follows. The calculations given in [19, 20] were performed in the semi classical limit $g_s \ll 1$. Looking at (2.11) we get that $\mu \gg 1$ and consequently the coefficient in front of the linear term in (2.11) is very large. To compare this result with the effective field theory analysis we must mention that the tachyon profile $T = (a + b\phi)e^{2\phi}$ is also solution of the equation of motion of the tachyon effective action (2.2) when the background dilaton and metric are given in (2.8). However we have shown in [18] that such a tachyon profile induces large backreaction on metric and dilaton and thus cannot be considered as an exact solution of the action $S = S_{g,\Phi} + S_T$. This fact has simple explanation. The proposal for the tachyon effective action (2.2) was based on previous works considering tachyon effective actions on unstable D-branes [12, 13, 14]. It was argued in [12, 13] that tachyon effective actions that have the tachyon marginal profile as their exact solution correctly capture tachyon dynamics very close to the marginal tachyon solution in the form $T \sim e^{\beta_\mu x^\mu}$. In 2D bosonic string theory, $\beta_1 = 2$ and for the tachyon solution $T = (a+b\phi)e^{2\phi}$ the condition that this profile is close to $T \sim e^{2\phi}$ implies that $b \ll 1$. Comparing with the (2.11) we see that this corresponds to $g_s \gg 1$ which is certainly out of region of validity of the classical analysis. This result also implies that the tachyon effective action (2.2) cannot be derived from the collective field theory that arises in the classical description of the matrix model [25, 26]. We must stress that remarks given above do not solve the problems how to relate (2.10) and (2.11). In order to do that we should find how the tachyon effective action (2.2) can be derived (We hope that this could be done.) from the microscopic formulation of 2D string theory. However in spite of these puzzling facts we still believe that the action (2.2) could capture some problems in 2D string theory that cannot be analysed in semi classical approximation.

[6] Very nice arguments based on the analysis of minisuperspace wave functions in the $c = 1$ models that support the claim that the vacuum expectation value of the tachyon is equal $T \sim \phi e^{2\phi}$ for $\phi \to -\infty$ were given in [21, 22, 23]. On the other hand it was also mentioned in [23] that the minisuperspace wave functions satisfy the Wheeler-de-Witt equation with the tachyon vacuum value $< T >= \mu e^{2\phi}$ rather then with $\langle T \rangle = \mu\phi e^{2\phi}$. It is not clear how this is consistent with the fact that the cosmological constant operator in $c = 1$ model should be $\phi e^{2\phi}$.

To support this claim we begin to study the action (2.2) in more details. Let us introduce the scalar field $S(x)$ through

$$T(\phi, t) = e^{2\phi} S(\phi, t) . \tag{2.12}$$

In terms of this field the tachyon effective action (2.2) in the linear dilaton background (2.8) takes simple form

$$S = -\int dt d\phi \left[\sqrt{1 - (\partial_t S)^2 + (\partial_\phi S)^2} - 1 \right] . \tag{2.13}$$

Now the equation of motion that arises from (2.13) is

$$\partial_\mu \left[\frac{\eta^{\mu\nu}\partial_\nu S}{\sqrt{1 + \eta^{\mu\nu}\partial_\mu S \partial_\nu S}} \right] = 0 . \tag{2.14}$$

The natural solution of this equation of motion is

$$S_c = \mu = \text{const.} . \tag{2.15}$$

Moreover, in terms of the field S the stress energy tensor and dilaton source have the form

$$T_{\mu\nu} = \left(-g_{\mu\nu} + 2S^2 V_\mu V_\nu + \partial_\mu S \partial_\nu S - S^2 V_\nu V_\mu \right) \sqrt{\mathbf{B}} + g_{\mu\nu} - S^2 V_\mu V_\nu \tag{2.16}$$

and

$$J_\Phi = 2\partial_\mu \left[S^2 g^{\mu\nu} V_\nu \sqrt{\mathbf{B}} \right] + g^{\mu\nu}\partial_\mu S \partial_\nu S \frac{1}{\sqrt{\mathbf{B}}} - \partial_\mu \left[g^{\mu\nu}(V_\nu S + \partial_\nu S) \frac{1}{\sqrt{\mathbf{B}}} \right] - \partial_\mu \left[S^2 g^{\mu\nu} V_\nu \right] , \tag{2.17}$$

where $\mathbf{B} = 1 + \eta^{\mu\nu}\partial_\mu S \partial_\nu S$. It is easy to see that components of the stress energy tensor (2.16) and the dilaton source (2.17) are equal to zero for $S_c = \mu$.

It is interesting to study the action for fluctuation modes around the classical solution S_c. If we write the general field S as

$$S(\phi, t) = S_c + s(\phi, t) \tag{2.18}$$

and insert it to the action (2.13) we obtain the action for $s(\phi, t)$ that is exactly the same as the original one. This can be regarded as a consequence of the fact that all values of parameter μ are equivalent.

In the end of this section we will briefly analyse another solution of the equation of motion (2.14) that has the form

$$S = a\phi + b . \tag{2.19}$$

However for such a tachyon profile the components of the stress energy tensor are equal to

$$\begin{aligned} T_{tt} &= 0 , \qquad T_{t\phi} = 0 , \\ T_{\phi\phi} &= 1 - \sqrt{1 + a^2} + 4(a\phi + b)^2(\sqrt{1 + a^2} - 1) . \end{aligned} \tag{2.20}$$

We see that $T_{\phi\phi}$ does not vanish. Then one can expect strong backreaction on the metric and hence the equation of motion of two dimensional gravity is not satisfied for $g_{\mu\nu} = \eta_{\mu\nu}$.

One can perform the same analysis for the dilaton source with the same conclusion. For that reason we will not consider the solution (2.19) further in this paper since its meaning in the context of the proposed model of 2D string theory is questionable.

In the next section we begin the analysis of the quantum properties of the action (2.13). We start with the simple example of the quantum mechanics of the zero mode.

3. Quantum mechanics of zero mode

We start the analysis of the quantum properties of the action (2.13) with the study of the dynamics of the zero mode S_0. In this approximation we neglect the dependence of the field S on the spatial coordinate ϕ. This approach resembles minisuperspace analysis well known from the study of two dimensional string theory [21] and which was recently used for analysis of the particle production on S-branes [28, 29].

To begin with let us consider the lagrangian for the zero mode S_0 in the form

$$L = -\sqrt{1 - \dot{S}_0^2} + 1 \,. \tag{3.1}$$

Then the hamiltonian is equal to

$$H = P\dot{S}_0 - L = \sqrt{1 + P^2} - 1 \,, \tag{3.2}$$

where we have used the fact that the conjugate momentum P is

$$P = \frac{\dot{S}_0}{\sqrt{1 - \dot{S}_0}} \,. \tag{3.3}$$

The quantisation of the system given above is straightforward. The basic commutation relation is

$$[\hat{S}_0, \hat{P}] = i \tag{3.4}$$

which implies that in the coordinate representation $\hat{P}$ has the standard form $P = -i\frac{\partial}{\partial S_0}$ when acts on the wave function $\Psi(S_0) = \langle S_0|\Psi\rangle$, where $|S_0\rangle$ is eigenvector of the operator $\hat{S}_0$:

$$\hat{S}_0 \left|S_0\right\rangle = S_0 \left|S_0\right\rangle \,, \qquad \left\langle S_0|S_0'\right\rangle = \delta(S_0 - S_0') \,. \tag{3.5}$$

From the analysis of the classical equation of motion we know that the tachyon configuration that does not induce any backreaction on the metric and dilaton corresponds to the hamiltonian that is equal to zero. Then it is natural to define the vacuum state as an eigenstate of the hamiltonian

$$\hat{H}\Psi(S_0) = E\Psi(S_0) \tag{3.6}$$

that has zero energy. Now (3.2) implies

$$\hat{P}^2 \left|\Psi_0\right\rangle = 0 \Rightarrow -\frac{\partial^2 \Psi_0(S_0)}{\partial^2 S_0} = 0 \tag{3.7}$$

that has general solution

$$\Psi_0(S_0) = \frac{1}{N}(aS_0 + b) \,, \tag{3.8}$$

where a, b are some constants and where N is normalisation factor. To clarify its meaning note that the coordinate S_0 takes values in the interval $(-\infty, \infty)$. Then the normalisation condition implies

$$1 = \int dS_0 |\Psi_0|^2 \Rightarrow N^2 = \frac{2a^2}{3} L^3 + 2b^2 L \,, \tag{3.9}$$

where we have introduced the IR cut-off L. We see that the wave function $\Psi_0(S_0)$ is not normalisable in the limit $L \to \infty$. On the other hand the expectation value of the operator $\hat{S}_0$ in the state $|\Psi_0\rangle$ is finite and it is equal to

$$\left\langle \Psi_0 | \hat{S}_0 | \Psi_0 \right\rangle = \int dS_0 \Psi_0(S_0) S_0 \Psi_0(S_0) = \frac{4abL^3}{3N^2} = \frac{2b}{a(1 + \frac{3b^2}{a^2 L^2})} \Rightarrow \frac{2b}{a} \equiv \mu \,, \quad \text{for } L \to \infty \,. \tag{3.10}$$

We must say few words to the fact that $\Psi_0(S_0)$ is not normalisable. Usually such functions are discarded from the Hilbert space of given theory. On the other hand we know from the analysis of two dimensional string theory that non normalisable states are important for description of given theory. More precisely, it was argued in [21, 23] that since these wave functions do not fluctuate they parametrise different backgrounds of given theory. As we will see more clearly in the next section the same conclusion can be said about the vacuum states given above.

Let us now consider following operator

$$\hat{O}_\epsilon = e^{-i\epsilon \hat{P}} \,, \tag{3.11}$$

where ϵ is real parameter. Now we are going to argue that this operator has similar properties as the spectral flow operator that was discussed in the context of two dimensional string theory in [24]. First of all, it is easy to prove following commutation relation

$$[\hat{S}_0, \hat{O}_\epsilon] = \epsilon \hat{O}_\epsilon \,. \tag{3.12}$$

Then we can consider the state

$$\left| \Psi_0' \right\rangle = \hat{O}_\epsilon \left| \Psi_0 \right\rangle \,. \tag{3.13}$$

Using (3.12) it is easy to calculate the expectation value of the operator $\hat{S}_0$ in$|\Psi'\rangle$

$$\left\langle \Psi_0' | \hat{S}_0 | \Psi_0' \right\rangle = \mu + \epsilon \,. \tag{3.14}$$

Using also the fact that $\hat{O}_\epsilon$ commutes with $\hat{H}$ it easy to see that $|\Psi'\rangle$ is eigenstate of the hamiltonian with zero energy and consequently it can be regarded as the new vacuum state characterised by the expectation value (3.14). In other words, if we choose one particular vacuum state $|\Psi_0\rangle$ with the expectation value $\left\langle \hat{S}_0 \right\rangle = \mu$ one can construct family of all vacuum states through the action of the operator $\hat{O}_\epsilon$ on $|\Psi_0\rangle$. We see that all these states are equivalent and differ in the expectation value of $\hat{S}_0$ and consequently can be considered as quantum analogues of the classical solutions (2.15).

For consistency of the zero mode quantum mechanics we should show that the vacuum states do not induce any backreaction on metric and dilaton. In order to see this we must

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calculate the vacuum expectation value of the zero mode truncation of the stress energy tensor and the dilaton source. As follows from (2.16) the zero mode truncation of the stress energy tensor is equal to

$$\hat{T}_{\mu\nu} = \left(-g_{\mu\nu} + 2\hat{S}_0^2 V_\mu V_\nu - \hat{S}_0^2 V_\nu V_\mu\right)\sqrt{1+\hat{P}_0^2} + g_{\mu\nu} - \hat{S}_0^2 V_\mu V_\nu \,. \tag{3.15}$$

For the spatial linear dilaton $V_\phi = 2$ we get

$$\begin{aligned} \hat{T}_{tt} &= \sqrt{1+\hat{P}^2} - 1\,, \\ \hat{T}_{\phi\phi} &= (-1+4\hat{S}_0^2)\sqrt{1+\hat{P}^2} + 1 - 4\hat{S}_0^2\,, \\ \hat{T}_{t\phi} &= 0\,. \end{aligned} \tag{3.16}$$

Since $\hat{T}_{tt}$ is the same as the hamiltonian (3.2) we immediately obtain that its vacuum expectation value is equal to zero. For $\hat{T}_{\phi\phi}$ the action of the square root on Ψ_0 is equal to Ψ_0 and hence the vacuum expectation value of $\hat{T}_{\phi\phi}$ is equal to zero as well. This result implies that there is not any backreaction of the vacuum states on the metric even at the quantum mechanical level.

Now we are going to calculate the vacuum expectation value of the zero mode truncation of the dilaton source (2.17) that is equal to

$$\hat{J}_\Phi = -\frac{\hat{P}^2}{(1+\hat{P}^2)^{3/2}} + \frac{d}{dt}\left[\hat{S}_0 \frac{\hat{P}}{1+\hat{P}^2}\right]. \tag{3.17}$$

We immediately see that the action of the first term on Ψ_0 is equal to zero. Slightly more complicated is the calculation of the expectation value of the second term in (3.17). It is clear that passing from classical to quantum expression there is potential ambiguity in ordering of S and P. We will adopt the ordering that the operator $\hat{P}$ is always in the right to the operator $\hat{S}_0$. In any case since the expectation value of the time derivative of any operator $\hat{A}$ is equal to

$$\langle\Psi|\,\frac{d\hat{A}}{dt}\,|\Psi\rangle = \langle\Psi|\left(\frac{\partial\hat{A}}{\partial t} + i[\hat{H},\hat{A}]\right)|\Psi\rangle \tag{3.18}$$

we see that the vacuum expectation value of the second term in (3.17) vanish as well due to the fact that $\hat{H}\,|\Psi_0\rangle = 0$ and the fact that the operator $\hat{S}_0\frac{\hat{P}}{1+\hat{P}^2}$ does not depend on time.

In order to obtain more precise picture of the vacuum states of (2.13) we should take into account the dynamics of the nonzero modes as well. This will be done in the next section.

4. Quantum mechanics of the tachyon effective action

We begin this section with the construction of the hamiltonian from lagrangian given in (2.13)

$$H = \int d\phi\,[\Pi\partial_t S - L] = \int d\phi\left[\sqrt{(1+(\partial_\phi S)^2)(1+\Pi^2)} - 1\right], \tag{4.1}$$

where $\Pi(\phi,t) = \delta\mathcal{L}/\delta\partial_t S(\phi,t)$ is momentum conjugate to $S(\phi,t)$. It turns out that it is convenient to split the fields S,Π into their zero mode parts and the fluctuation parts as

$$\begin{aligned}
S(\phi,t) &= S_0(t)+\psi(\phi,t)\,, \qquad S_0(t)=\frac{1}{V_\phi}\int d\phi S(\phi,t)\,, \qquad \int d\phi\psi(\phi,t)=0\,,\\
\Pi(\phi,t) &= \frac{P(t)}{V_\phi}+\pi(\phi,t)\,, \qquad P=\int d\phi\Pi(\phi,t)\,, \qquad \int d\phi\pi(\phi,t)=0\,,
\end{aligned} \tag{4.2}$$

where V_ϕ is regularized volume of spatial section. In quantum theory we regard the fields S,Π as quantum operators $\hat{S}(\phi,t)$, $\hat{\Pi}(\phi,t)$ with the standard equal time commutation relations

$$[\hat{S}(\phi,t),\hat{S}(\phi',t)]=[\hat{\Pi}(\phi,t),\hat{\Pi}(\phi',t)]=0\,, \qquad [\hat{S}(\phi,t),\hat{\Pi}(\phi',t)]=i\delta(\phi-\phi')\,. \tag{4.3}$$

Then we immediately get

$$[\hat{S}_0,\hat{P}]=\frac{1}{V_\phi}\int d\phi d\phi'[\hat{S}(\phi),\hat{\Pi}(\phi')]=\frac{i}{V_\phi}\int d\phi d\phi'\delta(\phi-\phi')=i \tag{4.4}$$

and also

$$[\hat{\psi}(\phi),\hat{P}]=[\hat{S}_0,\hat{\pi}(\phi)]=0\,. \tag{4.5}$$

As we have argued in previous sections it is believed that the tachyon effective action (2.2) is valid for description of almost homogeneous field S. Then it is natural to restrict ourselves in the hamiltonian (4.1) to the leading order terms in $\partial_\phi\psi,\pi$. With this approximation the quantum hamiltonian operator has the form

$$\hat{H}=V_\phi\left(\sqrt{1+\frac{\hat{P}^2}{V_\phi^2}}-1\right)+\frac{1}{2}\int d\phi\left(\frac{\hat{\pi}^2+(\partial_\phi\hat{\psi})^2}{\sqrt{1+\frac{\hat{P}^2}{V_\phi^2}}}+\frac{(\partial_\phi\hat{\psi})^2\hat{P}^2}{V_\phi^2\sqrt{1+\frac{\hat{P}^2}{V_\phi^2}}}\right). \tag{4.6}$$

Note that thanks to the commutation relations (4.5) and the fact that the hamiltonian does not contain zero mode operator $\hat{S}_0$ there are not any problems with the ordering of various operators in (4.6).

In order to find ground states of the hamiltonian (4.6) we will work in the Schrödinger representation of the quantum field theory. In this description the explicit time dependence of the system is expressed through the time-dependent wave functionals $\Psi(\phi,t)$. More precisely, the time dependence of the wave functional is determined by Schrödinger equation

$$i\frac{\partial\Psi(\phi,t)}{\partial t}=\hat{H}\left(-i\frac{\delta}{\delta S_0}\,,S_0,-i\frac{\delta}{\delta\psi},\psi\right)\Psi(\phi,t)\ , \tag{4.7}$$

where $\hat{H}$ is given in (4.6). Let us now presume that the vacuum wave functional has the form

$$\Psi(S_0,\psi)=\Psi_0(S_0)\Phi(\psi)\,, \tag{4.8}$$

where the zero mode part $\Psi_0(S_0)$ obeys

$$\sqrt{1+\frac{\hat{P}^2}{V_\phi^2}}\Psi_0(S_0)=\Psi_0(S_0)\,. \tag{4.9}$$

From (4.9) we see that the vacuum state $\Psi_0(S_0)$ is solution of the differential equation

$$\hat{P}^2\Psi_0(S_0) = -\frac{\partial^2\Psi_0(S_0)}{\partial S_0^2} = 0 \tag{4.10}$$

that is the same as the equation disused in the context of the quantum mechanics of the zero mode in the previous section. Consequently the vacuum wave functional $\Psi(S_0, \psi)$ is labelled with the parameter μ that is vacuum expectation value of the zero mode operator $\hat{S}_0$. It is also clear that we can define the operator $\hat{O}_\epsilon = e^{-i\epsilon\hat{P}}$ that maps the vacuum state with $\left\langle \hat{S}_0 \right\rangle = \mu$ to the state $\Psi(\phi, t)' = \hat{O}_\epsilon\Psi_0(S_0)\Phi(\phi, t)$. Since $\hat{P}$ commutes with $\hat{\psi}$, $\hat{\pi}$, $\hat{O}_\epsilon$ acts on the zero mode part of the vacuum wave functional only we immediately see that $\hat{O}_\epsilon$ is the same as the operator (3.11).

In order to determine the vacuum wave functional for nonzero modes we use the Schrödinger equation (4.7) that for (4.8) is equal to

$$\Psi_0(S_0)i\frac{\partial\Phi(\psi)}{\partial t} = \Psi_0(S_0)\frac{1}{2}\int d\phi \left(\hat{\pi}^2 + (\partial_\phi\hat{\psi})^2\right)\Phi(\psi)\,. \tag{4.11}$$

In the previous equation we have used the fact that

$$\begin{aligned}\hat{H}\Psi(S_0,\psi) &= \frac{1}{2}\int d\phi \left(\frac{\hat{\pi}^2 + (\partial_\phi\hat{\psi})^2}{\sqrt{1+\frac{\hat{P}^2}{V_\phi^2}}} + \frac{(\partial_\phi\hat{\psi})^2\hat{P}^2}{V_\phi^2\sqrt{1+\frac{\hat{P}^2}{V_\phi^2}}}\right)\Psi_0(S_0)\Phi(\psi) \\ &= \Psi_0(S_0)\frac{1}{2}\int d\phi\left(\hat{\pi}^2 + (\partial_\phi\hat{\psi})^2\right)\Phi(\psi)\end{aligned} \tag{4.12}$$

which is a consequence of (4.9). Hence $\Phi(\psi)$ obeys the equation

$$\begin{aligned} i\frac{\partial\Phi(\psi,t)}{\partial t} &= \frac{1}{2}\int d\phi\left(-\frac{\delta^2}{\delta^2\psi} + (\partial_\phi\psi)^2\right)\Phi(\psi) \Rightarrow \\ \Rightarrow i\frac{\partial\Phi(\psi)}{\partial t} &= \frac{1}{2}\int\frac{dk}{2\pi}\left(-\frac{\delta^2}{\delta\alpha(k)\delta\alpha(-k)} + k^2\alpha(k)\alpha(-k)\right)\Phi(\alpha(k))\,,\end{aligned} \tag{4.13}$$

where the second line in (4.13) is transformation of the first line to the momentum representation. The solution of (4.13) that corresponds to the ground state is well known [30, 31]. It can be shown that the vacuum wave functional is equal to

$$\Phi(\psi) = N(t)\exp\left[-\frac{1}{2}\int\frac{dk}{(2\pi)}\alpha(k)A(k)\alpha(-k)\right) \equiv N(t)E(\alpha(k))\,, \tag{4.14}$$

where

$$N(t) = C\exp\left[-i\frac{1}{2}\int\frac{dk}{(2\pi)}A(k)\right]\,, \qquad A(k) = |k|\,. \tag{4.15}$$

In other words the vacuum state for fluctuation is ordinary vacuum wave functional for free, massless particles in two dimensions. The time-dependent factor $N(t)$ can be written as $N(t) = e^{-iE_0t}$ where the vacuum energy of oscillators is $E_0 = \frac{1}{2}\int dk|k| = \Lambda^2/2$. The vacuum energy depends on the the cut-off Λ that expresses the fact that the tachyon effective action (2.13) is valid for tachyon fields close to the marginal tachyon profile $e^{2\phi}$. For fluctuation modes this condition implies that $\Lambda \ll 1$.

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Let us review results obtained in this section. We have got the family of vacuum states of the tachyon effective action that are labelled with the continuous parameter μ. We have argued that μ corresponds to the vacuum expectation value of the zero mode operator $\hat{S}_0$. We were also able to find operator $\hat{O}_\epsilon$ that maps any vacuum state labelled with μ to another vacuum state labelled with the parameter $\mu + \epsilon$. We mean that existence of this operators shows that all vacuum states with different $\langle \hat{S}_0 \rangle$ are equivalent. We can also alternatively interpret this result as the fact that vacuum states with different μ's parametrise superlescion sectors of given theory. We have also seen that the vacuum state for fluctuations is ordinary vacuum wave functional for massless scalar field in Minkowski spacetime with the exception that there is a natural cut-off Λ that arises from the limited region of validity of the tachyon effective action (2.13). Note that the vacuum wave functional for fluctuations does not depend on μ.

4.1 Calculation of the expectation values

In the previous section we have found family of vacuum states of the tachyon effective action in the linear dilaton background. However as in section (3) we must show that these states do not induce backreaction on metric and dilaton. For that reason we will now calculate the expectation values of the dilaton source and stress energy tensor in the state (4.8).

To begin with we split the field S into its zero mode and nonzero mode parts and insert them into (2.16) so that for $V_\phi = 2$ we get classical components of the stress energy tensor

$$\begin{aligned}
T_{tt}(\phi, t) &= (1 + \dot{S}_0^2 + 2(\partial_t \psi)\dot{S}_0 + (\partial_t \psi)^2)\sqrt{\mathbf{B}} - 1 \,, \\
T_{\phi\phi}(\phi, t) &= (-1 + 4S_0^2 + 8S_0\psi + 4\psi^2 - \partial_\phi \psi \partial_\phi \psi)\sqrt{\mathbf{B}} + 1 - 4S_0^2 - 8S_0\psi - 4\psi^2) \,, \\
T_{t\phi}(\phi, t) &= T_{\phi t}(\phi, t) = \partial_\phi \psi (\partial_t S_0 + \partial_t \psi)\sqrt{\mathbf{B}} \,.
\end{aligned} \tag{4.16}$$

As in previous section we will presume that $\partial_\phi \psi$, π are small so that $\mathbf{B}$ is equal to

$$\sqrt{\mathbf{B}} = \frac{1}{\sqrt{1 + \frac{P^2}{V_\phi^2}}} \left(1 + \frac{1}{2} \left[(\partial_\phi \psi)^2 - \frac{\pi^2}{1 + \frac{P^2}{V_\phi^2}} \right] \right) . \tag{4.17}$$

Then the quantum stress energy tensor can be written as

$$\begin{aligned}
\hat{T}_{tt} &= (1 + (\partial_t \hat{S}_0)^2 + 2\partial_t \hat{\psi} \partial_t \hat{S}_0 + (\partial_t \hat{\psi})^2) \frac{1}{\sqrt{1 + \frac{\hat{P}^2}{V_\phi^2}}} \left(1 + \frac{1}{2} \left[(\partial_\phi \hat{\psi})^2 - \frac{\hat{\pi}^2}{1 + \frac{\hat{P}^2}{V_\phi^2}} \right] \right) - 1 \,, \\
\hat{T}_{\phi\phi} &= (-1 + 4\hat{S}_0^2 + 8\hat{S}_0 \hat{\psi} + 4\hat{\psi}^2 - (\partial_\phi \hat{\psi})^2) \frac{1}{\sqrt{1 + \frac{\hat{P}^2}{V_\phi^2}}} \left(1 + \frac{1}{2} \left[(\partial_\phi \hat{\psi})^2 - \frac{\hat{\pi}^2}{1 + \frac{\hat{P}^2}{V_\phi^2}} \right] \right) + \\
&\quad +1 - 4\hat{S}_0^2 - 8\hat{S}_0 \hat{\psi} - 4\hat{\psi}^2 \,, \\
\hat{T}_{t\phi} &= \hat{T}_{\phi t} = \partial_\phi \hat{\psi} (\partial_t \hat{S}_0 + \partial_t \hat{\psi}) \frac{1}{\sqrt{1 + \frac{\hat{P}^2}{V_\phi^2}}} \left(1 + \frac{1}{2} \left[(\partial_\phi \hat{\psi})^2 - \frac{\hat{\pi}^2}{1 + \frac{\hat{P}^2}{V_\phi^2}} \right] \right) .
\end{aligned} \tag{4.18}$$

We would like to stress that for zero mode operators we adopt the same ordering of various operators as was defined in section (3) that means that $\hat{P}$ is always on the right with respect to $\hat{S}_0$. On the other hand we will argue that for nonzero modes the more natural is to define the stress energy tensor and the dilaton source as normal ordered quantities.

Now we can proceed to the calculation of the expectation value of the components of the stress energy tensor (4.18) in the vacuum state (4.8). As the first step we calculate the expectation value with respect to the zero mode part $\Psi_0(S_0)$. Since $\hat{P}^2\Psi_0(S_0) = 0$ we easily obtain

$$\widehat{\sqrt{\mathbf{B}}}\Psi(S_0,\psi) = \Psi_0(S_0)\left[1+\frac{1}{2}((\partial_\phi\hat{\psi})^2-\hat{\pi}^2)\right]\Phi(\psi)\,. \tag{4.19}$$

Now let us consider the time derivative of $\hat{S}_0$ that is equal to

$$\frac{d\hat{S}_0}{dt} = i[\hat{H},\hat{S}_0]\,. \tag{4.20}$$

The vacuum expectation value of this expression is zero since the vacuum state Ψ_0 is eigenstate of the hamiltonian. On the other hand we can calculate the right hand side of the equation (4.20) directly and we obtain that the commutator given there is proportional to $\hat{P}$. This result implies that $(\partial_t\hat{S}_0)^2 \sim \hat{P}^2$ and consequently its vacuum expectation value is equal to zero as well. Then the calculation of the vacuum expectation value of the zero mode operator is straightforward. When we also restrict ourselves to the terms of quadratic order in $\hat{\psi}$, $\hat{\pi}$ we obtain that the vacuum expectation value of the stress energy tensor equal to

$$\begin{aligned}
\langle T_{tt}(\phi,t)\rangle &= \frac{1}{2}\left\langle\hat{\pi}^2+(\partial_\phi\hat{\psi})^2\right\rangle_{\mathrm{fluct}} \\
\langle T_{\phi\phi}(\phi,t)\rangle &= (-1+4\mu^2)\frac{1}{2}\left\langle(\partial_\phi\hat{\psi})^2-\hat{\pi}^2\right\rangle_{\mathrm{fluct}} - \left\langle(\partial_\phi\hat{\psi})^2\right\rangle_{\mathrm{fluct}}\,, \\
T_{\phi t}(\phi,t) &= \left\langle\partial_\phi\hat{\psi}\hat{\pi}\right\rangle_{\mathrm{fluct}}\,,
\end{aligned} \tag{4.21}$$

where $\langle\ldots\rangle_{\mathrm{fluct}}$ means the vacuum expectation value with respect to the vacuum state of the fluctuation modes. In the previous expressions we have also used the fact that

$$\partial_t\hat{\psi} = i[\hat{H},\hat{\psi}] = \frac{\hat{\pi}}{\sqrt{1+\frac{\hat{P}^2}{V_\phi}}} \tag{4.22}$$

and consequently

$$\left\langle\partial_t\hat{\psi}\right\rangle = \langle\hat{\pi}\rangle_{\mathrm{fluct}}\,. \tag{4.23}$$

Following [30] it is now easy to calculate the vacuum expectation value in (4.21) and we get

$$\begin{aligned}
\langle T_{tt}(\phi)\rangle_{\mathrm{fluct}} &= \int\frac{dk}{2\pi}\frac{dk}{2\pi}e^{i\phi(k+k')}\frac{1}{2}\left\langle -kk'\alpha(k)\alpha(k')-\frac{\delta^2}{\delta\alpha(k)\alpha(k')}\right\rangle_{\mathrm{fluct}}\,, \\
\langle T_{\phi\phi}(\phi)\rangle_{\mathrm{fluct}} &= (-1+4\mu^2)\int\frac{dk}{2\pi}\frac{dk}{2\pi}e^{i\phi(k+k')}\frac{1}{2}\left\langle -kk'\alpha(k)\alpha(k')+\frac{\delta^2}{\delta\alpha(k)\alpha(k')}\right\rangle_{\mathrm{fluct}} -
\end{aligned}$$

$$-\int \frac{dk}{2\pi}\frac{dk}{2\pi}e^{i\phi(k+k')}\left\langle kk'\alpha(k)\alpha(k')\right\rangle_{\text{fluct}},$$

$$\langle T_{\phi t}(\phi)\rangle_{\text{fluct}} = -\int \frac{dk}{2\pi}\frac{dk'}{2\pi}e^{i\phi(k+k')}\left\langle ik\alpha(k)\frac{\delta}{\delta\alpha(k')}\right\rangle_{\text{fluct}}. \tag{4.24}$$

Using again the results given in [30]

$$\left\langle\alpha(k)\alpha(k')\right\rangle_{\text{fluct}} = 2\pi\frac{\delta(k+k')}{2|k|}, \qquad \left\langle\frac{\delta^2}{\delta\alpha(k)\alpha(k')}\right\rangle_{\text{fluct}} = -\frac{k^2}{2|k|}2\pi\delta(k+k') \tag{4.25}$$

we finally obtain

$$\langle T_{tt}(\phi)\rangle_{\text{fluct}} = \frac{\Lambda^2}{2}, \qquad \langle T_{\phi\phi}(\phi)\rangle_{\text{fluct}} = -\frac{1}{2}\Lambda^2, \qquad \langle T_{\phi t}(\phi)\rangle_{\text{fluct}} = 0. \tag{4.26}$$

We see that the stress energy tensor is diagonal and its components scale with the cut-off Λ. This is the same result as in standard quantum field theory calculation however the interpretation is different. In ordinary QFT there is not restriction on Λ so that when we remove the cut-off by taking $\Lambda \to \infty$ we obtain divergent contributions. One possibility how to avoid such a behaviour is to define field theory operators as normal ordered with respect to the vacuum state which means that their vacuum expectation value is zero. Even if the cut-off Λ in (4.26) has clear meaning since it express the finite region of validity of the tachyon effective action (2.2) it is natural to consider stress energy tensor as normal ordered. Then the vacuum expectation value of the normal ordered stress energy tensor is equal to zero. In context of 2D effective field theory this result implies that the vacuum states do not induce any backreaction on metric.

As a next step we will calculate the vacuum expectation value of the dilaton source

$$\begin{aligned}
\hat{J}_\Phi(\phi) &= 4\partial_\phi\left[(\hat{S}_0+\hat{\psi})^2\sqrt{\mathbf{B}}\right] - ((\partial_t\hat{S}_0)+\partial_t\hat{\psi})^2\frac{1}{\sqrt{\mathbf{B}}} - \\
&\quad -(\partial_\phi\hat{\psi})^2\frac{1}{\sqrt{\mathbf{B}}} - \partial_\phi\left[(2(\hat{S}_0+\hat{\psi})+\partial_\phi\hat{\psi})\frac{1}{\sqrt{\mathbf{B}}}\right] + \\
&\quad +\partial_t\left[\partial_t(\hat{S}_0+\hat{\psi})\frac{1}{\sqrt{\mathbf{B}}}\right] - 2\partial_\phi\left[(\hat{S}_0+\hat{\psi})^2\right].
\end{aligned} \tag{4.27}$$

We can proceed in the same way as in the calculation of the expectation value of the stress energy tensor. More precisely, we begin with the calculation of the expectation value of the zero mode wave function $\Psi_0(S_0)$. If we again restrict to the terms in the leading order in $\partial\hat{\psi}$, $\hat{\pi}$ we obtain

$$\begin{aligned}
\left\langle\hat{J}_\Phi\right\rangle &= 8\left\langle\hat{\psi}\partial_\phi\hat{\psi}\right\rangle_{\text{fluct}} - \left\langle(\partial_t\hat{\psi})^2\right\rangle_{\text{fluct}} + \frac{1}{2}\left\langle(\partial_\phi\hat{\psi})^2+\hat{\pi}^2\right\rangle_{\text{fluct}} - \\
&\quad -\left\langle\partial_\phi[2(\mu+\hat{\psi})+\partial_\phi\hat{\psi}]\right\rangle_{\text{fluct}} + \left\langle\partial_t^2\hat{\psi}\right\rangle_{\text{fluct}} - 2\left\langle\partial_\phi(\mu+\hat{\psi})^2\right\rangle_{\text{fluct}}.
\end{aligned} \tag{4.28}$$

This expression can be calculated in the same way as in case of the evaluation of the vacuum expectation values of the stress energy tensor. It is then easy to show that the vacuum expectation value of $\hat{J}$ is equal to zero.

Let us outline results obtained in this section. By explicit calculations of the vacuum expectation values of the normal ordered components of the stress energy tensor we have shown that they are equal to zero and consequently these vacuum states do not induce any backreaction on metric. The same conclusion holds in case of dilaton source as well. We mean that these results provide further support for the consistency of vacuum states of the tachyon effective action (2.2).

5. Conclusion

In this article we have studied the vacuum states of the model of two dimensional tachyon effective field theory that was proposed in [18]. The main goal was to find whether there exists the similarity between the ground states in the tachyon effective action and the vacuum states in the microscopic formulation of 2D theory in terms of free fermions. We mean that looking for this relation could be useful for the study of the non perturbative aspects of the 2D string theory. In fact, we have proposed the model of the tachyon effective action that has family of ground states that do not induce any backreaction on metric and dilaton. We have also found that there exists operator $\hat{O}_\epsilon$ that maps one ground state with parameter μ to another one with parameter $\mu + \epsilon$. In other words all ground states with different values of μ are equivalent.

We mean that it would be very interesting to find similar structure in the microscopic formulation of 2D theory since we know that the tachyon field arises from the bosonisation in the dual matrix model. In particular, it would be nice to find operators in the quantum field theory of free fermions that are related to the operators $\hat{S}_0$, $\hat{O}_\epsilon$ in the effective field theory description. We hope to return to this problem in the near future.

There are many problems and open questions that deserve to be studied further. One of the most serious one is to find how the effective field theory emerge from the matrix model without restriction to the semi classical approximation. We believe that the approach developed in [32]–[37] could be helpful in solving of this problem. Another open problem is to find effective field theory models of two dimensional OA and OB theories. We expect that construction of these models could be straightforward extension of the model given in this paper when we include terms containing the Ramond-Ramond field C into the effective action. It would be certainly very interesting to see whether one can find such effective actions that have backgrounds given in [38]–[42] as their exact solutions.

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